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Eigenvalue/Eigenvector Assignment Using Output Feedback

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SUMMARY

The problem of eigenvalue assignment in a linear time-invariant system using output feedback is considered. New sufficient conditions are derived to assign an almost arbitrary set of minimum $(n, m + r - 1)$ distinct eigenvalues where n, m, and r are the number of states, inputs, and outputs, respectively. These conditions precisely identify the class of systems where such an assignment is impossible. The synthesis technique also highlights the freedom in selection of closed-loop eigenvectors under output feedback. The utility of eigenvalue/eigenvector assignment in transient response shaping is illustrated by the design of a controller for the lateral dynamics of an aircraft.

INTRODUCTION

Control system design based on eigenvalue or pole assignment has received a great deal of attention in the literature. It is well known that for a controllable system, if state variable feedback is employed, all eigenvalues can be assigned (ref. 1). Also it is known that for multi-input systems, the feedback law assigning a given set of eigenvalues is not unique and that different control laws can yield identical eigenvalues while yielding radically different eigenvectors. Since the eigenvectors determine the influence of each eigenvalue on each state variable response, failure to use the multi-input design freedom fully may result in undesirable mode coupling and other poor transient behavior. For n-state feedback systems, it has been shown (ref. 2) that with m inputs, in addition to the assignment of all n eigenvalues, up to m entries in each eigenvector can be arbitrarily assigned. However, the problem of eigenvalue assignment using output feedback instead of state feedback has not yet been completely resolved. The problem of determining conditions under which all eigenvalues of a system can be arbitrarily assigned to a system under output feedback has been investigated in references 3 and 4. Bounds on the number of states, in terms of number of inputs, outputs, controllability, and observability indices are established for complete pole assignability. Reference 5 shows that for a system with r outputs, if $mr \geq n$ then the system is pole-assignable provided the feedback gain elements are allowed to be complex numbers. References 6 and 7 address the converse problem: given a controllable, observable system, how many eigenvalues can be arbitrarily assigned to the system. In general, it is concluded (ref. 7) that minimum $(n, m + r - 1)$ eigenvalues can "almost" always be assigned to the system using output feedback. The qualification "almost" was introduced to cover classes of systems where such an assignment is impossible. In effect, the analysis in reference 7 does not precisely determine the conditions under which $(m + r - 1)$ eigenvalues cannot be assigned to the system.

This report considers the problem of determining the number of eigenvalues assignable to a given system. By formulating an eigenvalue/eigenvector assignment problem, sufficient conditions required for the assignment of minimum $(n, m + r - 1)$ eigenvalues are derived. These conditions precisely identify

the class of systems which can be assigned only d eigenvalues, where maximum (m,r) $\leq$ d < (m + r - 1). The new formulation permits the development of an algorithm to assign (m + r - 1) eigenvalues. In addition, (r - 1) eigenvectors can be partially assigned with, at most, m entries in each vector arbitrarily chosen. In the event n > (m + r - 1), various synthesis alternatives to stabilize the system are also investigated since in this case all system eigenvalues cannot be assigned. The counter example of reference 7 is used to demonstrate the utility of the new sufficient conditions in identifying systems which cannot be assigned (m + r - 1) eigenvalues. Finally, the advantage of both eigenvalue and eigenvector assignments in response shaping is illustrated by designing a controller to meet the lateral handling qualities specifications for an aircraft.

SYMBOLS

Values are given in SI and U.S. Customary Units. Calculations were made in U.S. Customary Units.

A   system matrix

$A \in R^{n \times n}$ n × n   real matrix   A

a_y lateral acceleration, m/sec^2 (ft/sec^2)

B   input matrix

C   measurement matrix

c   measurement vector

$\bar{D}$ matrix used in equation (45)

$D^{(k)}$ matrices defined by equation (B4)

d,i,j,k,t   indices

e_j vector defined in notation (1) in appendix B

F   matrix defined by equation (9)

$f_j^{(k-1)}$ vector defined by step 3(a) of appendix B

G   matrix defined by equation (10)

$g_j^{(k)}$ vector defined by step 3(a) of appendix B

H_i matrices used in equation (23) and derived by substituting equation (6) into equation (7)

$h_j^{(k-1)}$ vector defined by step 3(a) of appendix B

I_{index} (index)th order identity matrix

2

$j \in \left\{ \Delta^{(k)} \right\}$ j is an element of index set $\Delta^{(k)}$

K feedback matrix

L_i matrices used in equation (23) and derived by substituting equation (6) into equation (7)

ℓ feedback vector defined by equation (17)

$M^{(k)}, M^{(k-1)}, M^{(1)}$ matrices defined by notation (6) in appendix B

m number of inputs

max   maximum value

min   minimum value

N matrix defined by equation (13)

n number of states

P matrix used in equation (46) and defined immediately afterward

p roll rate, deg/sec

$Q^{(k-1)}, Q^{(i)}$ matrices defined by equation (B3)

$Q^{(0)}$ matrix defined by notation (3) in appendix B

q feedback vector defined by equation (18)

r number of outputs

S matrix defined by equation (8)

s vector defined immediately prior to equation (20)

T_0, T_1 transformation matrices defined where used

u input vector defined by equation (2)

V modal matrix (matrix of eigenvectors)

$V^{(r)}$ matrix used in equation (12)

v, v_i eigenvectors

w vector used in equation (6); a partition of v

x state vector defined by equations (1)

$x \in R^n$ $n \times 1$ real vector x

y   output vector defined by equations (1)

$Z^{(r)}, Z^{(t)}$ matrices containing the vectors z_i; i = 1 to r and 1 to t, respectively

z   vector used in equation (6); a partition of v

β sideslip, deg

$\Delta^{(1)}, \Delta_t^{(1)}, \Delta_r^{(1)}, \Delta^{(k)}$ set of indices defined in appendix B

δ_a aileron angular deflection, deg

δ_k determinant defined in step 1 of appendix B

δ_r rudder angular deflection, deg

$\epsilon\lambda_k, \epsilon z_k$ perturbed quantities defined in step 1 and step 4 of appendix B, respectively

θ vector defined by notation (6) in appendix B

Λ_r, Λ_t diagonal eigenvalue matrices used in equations (12) and (B6), respectively

$\lambda, \lambda_i, \lambda_k$ eigenvalues

ξ, η, μ_1, μ_2 vectors defined by equations (21)

σ_k scalar defined by notation (4) in appendix B

ϕ bank angle, deg

$\dot{\psi}$ yaw rate, deg/sec

Superscripts:

-1   matrix inverse

$\hat{}, \tilde{}, -, *$ transformed quantity

    Upper case letters of the alphabet indicate matrices; matrix subscripts indicate partitioned quantities. Dot over a quantity denotes derivative with respect to time. Prime denotes transpose.

EIGENVALUE/EIGENVECTOR ASSIGNMENT FORMULATION

    Consider a linear, time invariant, multivariable, controllable, observable system

$$\left. \begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx \end{array} \right\} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^r$, and B and C are full rank; and for a non-trivial problem formulation, assume $m,r > 1$ and $m,r < n$. The problem is to find a control law of the form

$$u = Ky \tag{2}$$

in order to assign arbitrary eigenvalues for the closed-loop system. To indicate clearly the freedom available in the selection of closed-loop eigenvalues and eigenvectors under output feedback, the measurement matrix C is assumed to be in a special canonical form:

$$\left. \begin{array}{l} C = \begin{bmatrix} C_1 & : & C_2 \end{bmatrix} \\ \\ C_1 = \begin{bmatrix} I_t \\ \cdots \\ 0 \end{bmatrix} \\ \\ C_2 = \begin{bmatrix} 0 \\ \cdots \\ c \end{bmatrix} \\ \\ c = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix} \end{array} \right\} \tag{3}$$

Here, $t = r - 1$, $C_1 \in R^{r \times t}$, $c \in R^{1 \times n-t}$, and I_t denotes a tth order identity matrix. Appendix A details a procedure for reducing any system (C,A,B) to this special form.

The closed-loop system matrix $(A + BKC)$ after applying feedback law (2) satisfies

$$(A + BKC)v_i = \lambda_i v_i \qquad\qquad (i = 1, 2, \ldots, n) \tag{4}$$

where λ_i is the ith eigenvalue and v_i is the corresponding eigenvector. The eigenvalue/eigenvector assignment problem is to determine the number of eigenvalues in equation (4) that can be arbitrarily assigned and to determine the freedom available in the selection of the associated eigenvectors.

In order to see what freedom exists in the choice of eigenvectors, write equation (4) in partitioned form as

$$\left\{ \begin{bmatrix} A_{11} & . & A_{12} \\ & . & \\ . & . & . & . & . \\ & . & \\ A_{21} & . & A_{22} \end{bmatrix} + \begin{bmatrix} B_1 \\ . & . & . \\ B_2 \end{bmatrix} KC \right\} v_i = \lambda_i v_i \tag{5}$$

5

where $A_{11}, B_1 \in R^{m \times m}$ and B_1 is nonsingular. Since B is full rank, the nonsingularity of B_1 can be assured, if necessary, by reordering the state variables in equations (1). Completing the multiplication of the partitioned matrices and some algebraic operations (ref. 8) permits equation (5) to be expressed as a set of constraints on the selection of eigenvectors. For clarity of presentation, these relations are detailed only for real eigenvalues. Extension to complex conjugate pairs in quasi-diagonal form yielding real eigenvector pairs is straightforward (ref. 8).

For real eigenvalues

$$\left[\lambda I_{n-m} - F \right] w = \left[G + \lambda S \right] z \tag{6}$$

$$\left[A_1 + B_1 KC \right] v = \lambda z \tag{7}$$

where λ is the eigenvalue, $v' = \left[z' : w' \right]$; v is the eigenvector with $z \in R^m$, and

$$S = B_2 B_1^{-1} \tag{8}$$

$$F = A_{22} - SA_{12} \tag{9}$$

$$G = A_{21} - SA_{11} \tag{10}$$

$$A_1 = \left[A_{11} : A_{12} \right] \tag{11}$$

Equation (6) represents an underdetermined system of $n - m$ equations in n unknowns. Thus m eigenvector entries corresponding to the z-vector can be chosen arbitrarily provided λ does not coincide with the spectrum of F. Examination of equation (7) reveals that at least r eigenvalues and r eigenvectors satisfying equation (6) can be assigned to the system in equations (1) by the feedback matrix

$$\bar{K} = B_1^{-1} \left[Z^{(r)} \Lambda_r - A_1 V^{(r)} \right] \left[CV^{(r)} \right]^{-1} \tag{12}$$

where Λ_r is the diagonal matrix of r eigenvalues and $Z^{(r)}$ and $V^{(r)}$ have the form $T^{(r)} = \left[t_1 : t_2 : \ldots : t_r \right]$ (where t_i are vectors). The solution to equation (12) is guaranteed provided the eigenvalues/eigenvectors are chosen to insure the nonsingularity of $\left[CV^{(r)} \right]$. It should be noted that in the case of state variable feedback ($C = I_n$), all n eigenvalues can be assigned to the system provided the modal matrix (matrix of eigenvectors) $V = \left[v_1 : v_2 : \ldots : v_n \right]$ is nonsingular. An algorithm which constructs such a nonsingular V is detailed in reference 8. Appendix B extends this algorithm to guarantee the nonsingularity of $\left[CV^{(r)} \right]$.

Notice that by carrying out this analysis on the dual system (B', A', C'), it can be shown that m eigenvalues can be assigned to the system. This analysis yields the following well-known result (ref. 9).

6

Lemma 1: For system (C,A,B), max(m,r) eigenvalues can be assigned using output feedback.

The analysis so far indicates that only max(m,r) eigenvalues can be assigned to the system using output feedback. However, by sacrificing some degree of freedom in the selection of the associated eigenvectors it is possible to extend the number of eigenvalues that can be assigned to min(n,m + r - 1) as is shown in the following section.

ALGORITHM TO ASSIGN MINIMUM (n,m + r - 1) EIGENVALUES

The basic approach in the development of this algorithm is to construct the output feedback law in equation (2) as a sum of two feedbacks $(\tilde{K} + \hat{K})$. The first feedback $(\tilde{K})$ assigns t eigenvalues, and the second feedback assigns additional min(m,n - t) eigenvalues while ensuring the protection of the t eigenvalues already assigned. The construction procedure yields a set of sufficient conditions for assigning min(n,m + r - 1) eigenvalues. These conditions also help characterize the class of systems which cannot be assigned min(n,m + r - 1) eigenvalues. Finally, some design freedom still exists to partially assign (r - 1) eigenvectors.

Step 1:

Assign t eigenvalues and corresponding eigenvectors to the system in equations (1) and form the matrix

$$N = \begin{bmatrix} N_1 \\ \cdots \\ N_2 \end{bmatrix} = \begin{bmatrix} v_1 & : & v_2 & : & \cdots & : & v_t \end{bmatrix} \tag{13}$$

with $N_1 \in R^{t \times t}$ and nonsingular. Appendix B details a procedure to construct N. Let $\tilde{K}$ be the nonunique feedback (eq. (B6)) corresponding to this assignment. Then the closed-loop matrix is

$$\tilde{A} = A + B\tilde{K}C \tag{14}$$

Step 2:

Apply a coordinate transformation

$$(C,\tilde{A},B) \rightarrow (CT_1, T_1^{-1}\tilde{A}T_1, T_1^{-1}B)$$

where

$$T_1 = \begin{bmatrix} N_1 & \cdot & 0 \\ & \cdot & \\ \cdots & \cdots & \cdots \\ & \cdot & \\ N_2 & \cdot & I_{n-t} \end{bmatrix} \tag{15}$$

The transformed system has the form

$$\dot{\hat{x}} = \begin{bmatrix} \Lambda_t & . & \hat{A}_{12} \\ & . & \\ \cdot\cdot\cdot & . & \cdot\cdot\cdot \\ & . & \\ 0 & . & \hat{A}_{22} \end{bmatrix} \hat{x} + \begin{bmatrix} \hat{B}_1 \\ \cdot\cdot\cdot \\ \hat{B}_2 \end{bmatrix} u \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \\ \end{matrix}\right\}$$

$$y = \begin{bmatrix} \hat{C}_1 & : & C_2 \end{bmatrix} \hat{x}$$

(16)

where $\Lambda_t \in R^{t \times t}$ is the diagonal matrix of eigenvalues assigned in step 1. It should be noted that C_2 is invariant under the transformation T_1.

Step 3:

In order to assign additional eigenvalues to the system of equations (16) while protecting t eigenvalues (Λ_t) already assigned, the second feedback ($\hat{K}$) is restricted to be of unity rank of the form $\hat{K} = q\ell'$ with $q \in R^m$, $\ell \in R^r$, and ℓ is chosen so that

$$\ell' \begin{bmatrix} \hat{C}_1 & : & C_2 \end{bmatrix} = \begin{bmatrix} 0 & : & c \end{bmatrix}$$

(17)

Now q must be chosen so that

$$\begin{bmatrix} \hat{A}_{22} + \hat{B}_2 q c \end{bmatrix}$$

(18)

is assigned $\min(m, n - t)$ eigenvalues.

Since $(\hat{A}_{22}, \hat{B}_2)$ is controllable, the following result holds:

Theorem 1: The single output subsystem $(c, \hat{A}_{22}, \hat{B}_2)$ can be assigned $\min(m, n - t)$ eigenvalues if and only if (a) $(c, \hat{A}_{22})$ is observable and (b) $\hat{B}_2$ is full rank.

Theorem 1 follows directly from lemma 1. Further, conditions (a) and (b) restrict the admissible set of eigenvalue/eigenvector assignments in step 1. For conceptual convenience, these parametric restrictions are formulated in terms of controllability conditions of a fictitious dynamic system in the state variable representation.

After matrix operations in equations (14) and (16) have been performed, the submatrix $\hat{A}_{22}$ can be written as

$$\hat{A}_{22} = \begin{bmatrix} A_{22} + B_2 \tilde{K} C_2 \end{bmatrix} + \tilde{N}_2 \begin{bmatrix} A_{12} + B_1 \tilde{K} C_2 \end{bmatrix}$$

(19)

where $\tilde{N}_2 = -N_2 N_1^{-1}$. Transposing equation (19) and writing $\tilde{K} C_2 = sc$, $s \in R^m$ yields

$$\hat{A}'_{22} = A'_{22} + c's'B'_2 + A'_{12}\tilde{N}'_2 + c's'B'_1\tilde{N}'_2$$

(20)

8

Equation (20) can now be looked upon as a system matrix derived by applying feedback to the dynamic system

$$\dot{\xi} = A_{22}'\xi + A_{12}'\mu_1 + c'\mu_2$$
$$\eta = B_2'\xi + B_1'\mu_1$$

(21)

with $\xi \in R^{n-t}$, $\eta \in R^m$, $\mu_1 \in R^t$, $\mu_2 \in R^1$, $\mu_1 = \tilde{N}_2'\xi$, $\mu_2 = s'\eta$, and $\hat{N}_2 = -N_2 N_1^{-1}$.

From equations (21) it can be shown that conditions (a) and (b) of theorem 1 are equivalent to the following conditions: (c) $\hat{B}_2 = B_2 + \tilde{N}_2 B_1$ and is full rank; (d) $\left\{ \left[A_{22}' + A_{12}'\tilde{N}_2' \right], c' \right\}$ is controllable. Conditions (c) and (d) clearly indicate the restriction on selection of the eigenvector parameters $\tilde{N}_2$ in step 1. This yields the following sufficient conditions:

> Theorem 2: The system (C, A, B) can be assigned $\min(n, m + r - 1)$ eigenvalues arbitrarily close to the desired set if the first t eigenvalues and eigenvectors in step 1 are chosen so that
>
> I N_1 is nonsingular
>
> II $\hat{B}_2 = B_2 + \tilde{N}_2 B_1$ and is full rank
>
> III $\left\{ \left[A_{22}' + A_{12}'\tilde{N}_2 \right], c' \right\}$ is controllable

"Arbitrarily close" in theorem 2 indicates that slight perturbations in eigenvalue specifications may be needed in the following situations:

(i) Assigned eigenvalues coincide with the spectrum of F (appendix B).

(ii) An exact combination of eigenvalue/eigenvector specifications in step 1 may not yield a nonsingular N_1 (appendix B).

(iii) Coincident spectrum situation similar to situation (i) exists for the subsystem eigenvalue assignment of theorem 1.

Condition I is required to guarantee the existence of the transformation T_1 in step 2. This condition can be explicitly included in the synthesis procedure as detailed in appendix B. Condition III is obtained from the property that for system (eqs. (21)), the class of feedback from input μ_1 should be restricted so that the controllability of the feedback system with respect to the input μ_2 is preserved. Conditions II and III yield nonlinear algebraic constraints for the elements of the eigenvector parameter matrix $\tilde{N}_2$ and thus, in general, can only be used as test conditions for each assignment in step 1. However, example 1 of the section entitled "Numerical Examples" shows how these conditions can be explicitly checked.

9

Step 4:

If theorem 2 holds, $\min(m, n - t)$ eigenvalues can be assigned to the single output system $(c, \hat{A}_{22}, \hat{B}_2)$ by the feedback q derived from lemma 1.

Step 5:

The composite feedback law

$$u = (\tilde{K} + \hat{K})y \tag{22}$$

assigns $\min(n, m + r - 1)$ eigenvalues to the system.

ASSIGNING n EIGENVALUES

The development so far has revealed that for systems where $n > (m + r - 1)$, all system eigenvalues cannot usually be assigned. However, by using equations (6) and (7), it is possible to derive conditions for assigning all eigenvalues of the system as follows.

If the eigenvalues to be assigned are noncoincident with the spectrum of F (if necessary, by a perturbation in specification), then equation (6) can be explicitly solved for the w-vector and substituted into equation (7). This substitution yields a set of homogeneous equations of the form

$$\left[H_i + B_1 K L_i\right] z_i = 0 \qquad (i = 1, 2, \ldots, n) \tag{23}$$

where H_i and L_i are readily derived. Then equation (23) has a nontrivial solution if and only if

$$\text{rank } \left[H_i + B_1 K L_i\right] < m \quad \text{for all } i \tag{24}$$

A set of n nonlinear equations in the m, r parameters of the gain matrix K can be derived by setting the appropriate determinant in equation (24) equal to zero. However, no general conclusions can be drawn regarding the existence of the solution.

An alternative approach not involving solution of nonlinear equations is to assign t eigenvalues as in equation (14) and to attempt to assign the remaining $(n - t)$ eigenvalues approximately to the subsystem in equation (18) using the condition in equation (24). In this case, a set of $(n - t)$ linear equations in m unknowns results, and a least-squares solution can be obtained. The foregoing discussions assume that $r \geq m$ if necessary by considering the dual system.

NUMERICAL EXAMPLES

Example 1

Consider the system described in reference 7 where

$$\left.\begin{array}{c}
\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\[20pt]
\bar{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \\[20pt]
\bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\end{array}\right\} \tag{25}$$

Reduce the system (25) to the special form of equations (1) by ordering state variables as (x_1, x_3, x_2, x_4) to make $\bar{C}_2$ nonsingular (appendix A) and by applying the coordinate transformation $(\bar{C}, \bar{A}, \bar{B}) \rightarrow (\bar{C}T_0, T_0^{-1}\bar{A}T_0, T_0^{-1}\bar{B})$ where

$$T_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

to yield

$$
A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 & 0 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}
$$

(27)

From equation (6) it can be shown that the closed-loop eigenvectors satisfy

$$
\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} n_2 \\ n_1 \end{bmatrix} = \begin{bmatrix} -\lambda & (1-\lambda) \\ 1 & 0 \end{bmatrix} \begin{bmatrix} n_3 \\ n_4 \end{bmatrix}
$$

(28)

where the eigenvalue is λ and the eigenvector is

$$
v = (n_1 \quad n_2 \quad n_3 \quad n_4)'
$$

(29)

From theorem 2, condition I, $n_1 \neq 0$; therefore, choose $n_1 = 1$ without loss of generality. Then, $\tilde{N}_2 = (-n_2 \quad -n_3 \quad -n_4)'$. Condition II is met for all choices of $\tilde{N}_2$ since $B_1' = 0$. Condition III implies that

$$
\left\{ \begin{bmatrix} 0 & 0 & 0 \\ -n_2 & -n_3 & -n_4 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}
$$

(30)

should be controllable. For the controllability matrix in equation (30) to be of full rank, the eigenvector parameters in equation (28) must satisfy

$$
n_2 n_3 - n_4 + n_3^2 + n_3 n_4 \neq 0
$$

(31)

By direct substitution equation (31) is seen to be violated by all admissible selections in equation (28). Thus, the system cannot be assigned $(m + r - 1) = 3$ eigenvalues. Indeed, only two eigenvalues can be assigned to

12

this system (ref. 7). However, the previous analyses (refs. 3 and 7) do not precisely lead to this conclusion.

Example 2

Consider the system described by (ref. 7)

$$
\left.
\begin{aligned}
\bar{A} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\[2em]
\bar{B} &= \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \\[2em]
\bar{C} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\end{aligned}
\right\}
\tag{32}
$$

It is required to assign eigenvalues close to -1, -2, and -5. The system (32) can be reduced to the form of equations (1) by following the procedure in appendix A as

Step 1: Measurement matrix is in the desired form with $\bar{C}_a$ nonsingular.

Step 2: Apply coordinate transformation

$$(\bar{C},\bar{A},\bar{B}) \rightarrow (\bar{C}T_0, T_0^{-1}\bar{A}T_0, T_0^{-1}\bar{B})$$

where

$$
T_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}
\tag{33}
$$

yielding

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix} \Bigg\}$$ (34)

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

The system (34) is in the required canonical form of equations (1). The closed-loop eigenvector constraints in equations (34) can be derived by identifying the respective matrices in equations (8) to (10) as

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$S = \begin{pmatrix} 1 & -1 \end{pmatrix}$$

$$F = 0$$

$$G = \begin{pmatrix} 0 & -1 \end{pmatrix}$$

and equation (6) yields

$$\lambda w = \begin{bmatrix} \lambda & : & -(1 + \lambda) \end{bmatrix} \begin{bmatrix} z_1 \\ \cdots \\ z_2 \end{bmatrix}$$ (35)

Equation (35) indicates that z_1 and z_2 can be arbitrarily chosen, provided $\lambda \neq 0$. Now, applying the algorithm to assign $(m + r - 1)$ eigenvalues yields the following synthesis sequence.

Step 1:

Assign $\lambda = -1$. From appendix B, case II, $\Delta_t{}^{(1)} = \{1\}$, and this implies $z_1 \neq 0$ in equation (35). (Condition I, theorem 2.) One acceptable assignment is

14

$$N = (1 \quad 1 \quad 1)'$$
(36)

and a nonunique feedback gain corresponding to this assignment from equation (B6) is

$$\tilde{K} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$$
(37)

The closed-loop system matrix of equation (14) is

$$\tilde{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$
(38)

Step 2:

Transform the system to the canonical form of equations (16) using

$$T_1 = \begin{bmatrix} 1 & . & 0 & 0 \\ & . & & \\ . & . & . & . & . & . \\ & . & & \\ 1 & . & 1 & 0 \\ & . & & \\ 1 & . & 0 & 1 \end{bmatrix}$$
(39)

to yield

$$\hat{A} = \begin{bmatrix} -1 & . & 0 & 0 \\ & . & & \\ . & . & . & . & . & . \\ & . & & \\ 0 & . & -1 & 0 \\ & . & & \\ 0 & . & 0 & 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} 1 & 0 \\ . & . & . & . \\ -1 & -1 \\ 0 & 1 \end{bmatrix}$$
(40)

$$\hat{C} = \begin{bmatrix} 1 & . & 0 & 0 \\ & . & & \\ 2 & . & 1 & 1 \end{bmatrix}$$

From equations (21) we can form the dynamic system

$$\dot{\xi} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \xi + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mu_1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mu_2$$

$$\eta = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \xi + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mu_1$$

(41)

with

$$-\mu_1 = (1 \quad 1)\xi, \quad \mu_2 = (-1 \quad 1)\eta$$

Since feedback from μ_1 does not affect the controllability of system (41) with respect to μ_2, the subsystem is pole assignable from condition III of theorem 2. Further, the assignment in step 1 satisfies condition II since $\hat{B}_2$ is nonsingular. Thus theorem 2 holds.

Step 3:

Choose $\ell' = (-2 \quad 1)$. This choice protects $\lambda = -1$ assigned in step 1. It now remains to choose q so that $\hat{K} = q\ell'$ assigns the eigenvalues $\lambda_2 = -2$ and $\lambda_3 = -5$.

Step 4:

Assign $\lambda_2 = -2$, $\lambda_3 = -5$ to the single output subsystem in equation (18) where

$$\hat{A}_{22} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\hat{B}_2 = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$$

(42)

$$c = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

16

with feedback

$$q = \begin{bmatrix} 6 \\ -10 \end{bmatrix} \Bigg\} \qquad (43)$$

$$\hat{K} = \begin{bmatrix} 6 \\ -10 \end{bmatrix} \begin{bmatrix} -2 & 1 \end{bmatrix} \Bigg\}$$

Step 5:

The feedback law of the form of equation (22) is given by

$$u = \left\{ \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -12 & 6 \\ 20 & -10 \end{bmatrix} \right\} y \qquad (44)$$

and assigns the eigenvalues -1, -2, and -5 to the system (eq. (32)).

AIRCRAFT LATERAL CONTROL DESIGN

The advantage of combined control of closed-loop eigenvalues and eigenvectors using state variable feedback has been investigated in reference 10. The utility of the output feedback extensions developed in this report will now be illustrated through the design of a lateral controller for an aircraft.

The linear perturbation model for the lateral motions of an aircraft can be modeled as

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{B}u \\ y &= \bar{C}x + \bar{D}u \end{aligned} \Bigg\} \qquad (45)$$

where x is the state vector of roll rate p, yaw rate $\dot{\psi}$, sideslip β, and bank angle ϕ, respectively. The control vector of aileron δ_a and rudder δ_r angular deflections is u. Roll rate p, yaw rate $\dot{\psi}$, and lateral acceleration a_y constitute the output vector y. All angles are in degrees, rates in deg/sec, and acceleration in m/sec^2 (ft/sec^2).

The respective matrices in equations (45) for a fighter aircraft at an altitude of 6096 m (20 000 ft), a Mach number of 0.67, and an angle of attack of 3.45° are given by

$$\bar{A} = \begin{bmatrix} -3.79 & 0.04 & -52 & 0 \\ -0.14 & -0.36 & 4.24 & 0 \\ 0.06 & -1 & -0.27 & 0.05 \\ 1 & 0.06 & 0 & 0 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 25 & 9.83 \\ 1.42 & -4.2 \\ 0.01 & 0.05 \\ 0 & 0 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -0.13 & -0.06 & -3.42 & 0 \end{bmatrix}$$

$$\bar{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.03 & -0.27 \end{bmatrix}$$

where the elements of the matrices are approximated to two significant digits.

Since the output vector in equations (45) is derived as a linear combination of both state variables and control inputs, the closed-loop system after applying feedback $u = K^*y$ takes the form

$$\left. \begin{aligned} x &= (\bar{A} + \bar{B}K\bar{C})x + \bar{B}Pu \\ y &= (\bar{C} + \bar{D}K\bar{C})x + \bar{D}Pu \end{aligned} \right\} \tag{46}$$

where $P = \left[I_m - K^*\bar{D}\right]^{-1}$ and $K = PK^*$ is the equivalent output feedback matrix obtained by setting $\bar{D} = 0$. Thus, the algorithms developed earlier for systems with $\bar{D} = 0$ are applicable to systems of the form of equations (45), provided P exists.

Then the feedback gain K^* is computed by using the relation

$$K^* = K\left[I_k + \bar{D}K\right]^{-1} \tag{47}$$

18

Direct matrix manipulations show that the inverse in equation (47) exists if P exists.

The handling qualities specifications (ref. 11) imply that the lateral aircraft dynamics should be composed of two weakly coupled subsystems. Roll rate and bank angle constitute the first subsystem and display predominantly the roll subsidence and spiral modes. The second subsystem is characterized by a well-damped Dutch roll mode defining the yaw rate and sideslip motions. These specifications can now be formulated as an eigenvalue/eigenvector assignment problem.

Table I summarizes the modal characteristics of the free aircraft. From the table it is seen that the Dutch roll mode is very lightly damped and appears dominantly in the response of the roll variables p and ϕ as evidenced by dominant entries in the corresponding eigenvector pair. Thus, the eigenvalue/eigenvector modification requires that the closed-loop system have the modes and mode-variable associations of table II. (In tables I to III, $j = \sqrt{-1}$.)

TABLE I.- MODAL CHARACTERISTICS OF FREE AIRCRAFT

Eigenvector components	Eigenvalue of -		
	-3.70 (Roll subsidence)	-0.35 ± j2.66 (Dutch roll)	-0.03 (Spiral)
p	-0.964	-0.403 0.829	-0.032
$\dot{\psi}$	-.041	-.096 -.131	.044
β	-.002	.069 -.034	.002
ϕ	.261	.324 .112	.998

TABLE II.- DESIRED MODAL SPECIFICATIONS

Mode	Eigenvalue	Dominant response variable
Roll subsidence	-6	Roll rate
Dutch roll	-1.0 ± j0.2	Yaw rate and sideslip
Spiral	-0.01	Bank angle

The output feedback analysis in the main text shows that all system eigen-values can be assigned since $n = m + r - 1$ and only two eigenvectors (t) can be assigned with at most two (m) entries in each vector arbitrarily chosen. The eigenvector freedom available was used to control the structure of the eigenvec-tor pair corresponding to the Dutch roll mode to effect the desired yaw rate and sideslip dominance. The modal coupling matrices $D^{(k)}$ (eq. (B4)) aid in the selection of the Dutch roll mode to yield the appropriate eigenvector forms. The four eigenvector entries that were freely chosen corresponded to the roll rate and yaw rate components of the real eigenvector pair associated with the Dutch roll mode. Since gain magnitude constraints cannot be explicitly included into the synthesis algorithm, the design parameters have to be iteratively modi-fied to meet gain limit requirements. After some design iterations, a compro-mise design yielded the modal characteristics summarized in table III. For example, it was noted that the Dutch roll mode damping could not be reduced (to improve ψ and β responses) without violating feedback gain limits which were set at unity for this analysis. The design yielded a feedback gain matrix K^{*} (eq. (47)) as

$$K^{*} = \begin{bmatrix} -0.19 & -0.2 & -0.66 \\ -0.16 & 0.19 & -0.6 \end{bmatrix}$$

TABLE III.- MODAL CHARACTERISTICS OF FEEDBACK AUGMENTED AIRCRAFT

Eigenvector components	Eigenvalue of -			
	-6 (Roll subsidence)	-1.0 ± j0.20 (Dutch roll)		-0.011 (Spiral)
p	0.986	0.01	0.02	-0.013
$\dot{\psi}$	.009	.35	.53	-.031
β	-.007	.25	.72	-.082
ϕ	-.165	-.05	.05	-.996

Table III illustrates that the closed-loop eigenvectors have approached the desired mode-decoupled structure. In particular, the desired modification achieved in the eigenvector pair corresponding to the Dutch roll mode should be noted. The improvement in transient response characteristics using the feedback controller is illustrated in figure 1. The response curves demonstrate that the cross coupling between the roll axis (p,ϕ) and yaw axis ($\dot{\psi}$,β) has been signifi-cantly reduced.

CONCLUDING REMARKS

New sufficient conditions to assign minimum $(n, m + r - 1)$ eigenvalues by means of output feedback have been derived. In general, in addition to the assignment of minimum $(n, m + r - 1)$ eigenvalues, $(r - 1)$ eigenvectors can be partially assigned with at most m entries in each vector arbitrarily chosen. The utility of assigning both eigenvalues and eigenvectors for response modification is illustrated by designing a feedback controller for the lateral dynamics of an aircraft. The synthesis algorithm is computationally simple and involves only the solution of a linear system of equations.

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APPENDIX A

REDUCTION OF (C,A,B) TO CANONICAL FORM

Consider the given system of the form

$$\left. \begin{array}{l} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \\ y = \bar{C}\bar{x} \end{array} \right\} \tag{A1}$$

Step 1:

Reorder the state variables (if necessary) so that the measurement matrix $\bar{C}$ is of the form

$$\bar{C} = \left[\bar{C}_a \; : \; \bar{C}_b \right] \tag{A2}$$

where $\bar{C}_a \in R^{r \times r}$ and nonsingular. It should be noted that the dimensions of the partitioned matrices in equation (A2) are different from those in equations (3).

Step 2:

Apply a coordinate transformation

$$\bar{x} = T_0 x \tag{A3}$$

where

$$T_0 = \begin{bmatrix} \bar{C}_a^{-1} & . & -\bar{C}_a^{-1}(\bar{C}_b - \bar{C}_s) \\ & . & \\ . & . & . & . & . & . & . & . & . & . \\ & . & \\ 0 & . & I_{n-r} \end{bmatrix} \tag{A4}$$

and $\bar{C}_s \in R^{r \times n-r}$ and $\bar{C}_s = \begin{bmatrix} 0 \\ . & . & . \\ c \end{bmatrix}$ with c as defined in equations (3). The transformed system

$$\left. \begin{array}{l} \dot{x} = (T_0^{-1}\bar{A}T_0)x + T_0^{-1}\bar{B}u \\ y = \bar{C}T_0 x \end{array} \right\} \tag{A5}$$

has the desired form of equations (1).

APPENDIX B

EIGENVALUE/EIGENVECTOR SELECTION PROCEDURE

For completeness of presentation, an existing algorithm (ref. 8) developed for eigenvalue/eigenvector assignment using state variable feedback is detailed in this appendix. Extensions are made to adopt the algorithm for the output feedback cases discussed in the main text.

SPECTRAL SYNTHESIS ALGORITHM

A direct way of constructing a nonsingular modal matrix is to generate the eigenvectors which satisfy equation (6) sequentially and insure that they do not lie in the eigenspace generated by the vectors already synthesized. The algorithm presented accomplishes this construction while constantly testing to insure that the set of eigenvectors is a linearly independent set to a degree determined by a numerical tolerance parameter set by the designer. For clarity of presentation the algorithm is detailed for real eigenvalue assignments. The following notations are used throughout the algorithm presentation:

(1) e_j is an n-vector with jth entry equal to 1 and all other entries equal to zero.

(2) $v_k' = \begin{bmatrix} z_k' & : & w_k' \end{bmatrix}$ is the kth eigenvector, where z_k is designer specified.

(3) $v_k^{(k-1)} = Q^{(k-1)}V_k$, where $Q^{(0)} = I_n$ and $Q^{(i)}$, $i \neq 0$, is defined in (7) which follows.

(4) $V_{jk}^{(k-1)}$ is the jth entry of $V_k^{(k-1)}$ and for convenience of notation is denoted σ_k.

(5) $\Delta^{(1)}$ is the set of indices (1, 2, . . ., n).

(6) $M^{(k)}$, an elementary upper triangular matrix of order n and index 1, takes the form

$$M^{(k)} = I_n - \theta e_j' \tag{B1}$$

with $e_j'\theta = 0$ and θ is chosen so that

$$M^{(k)}v_k^{(k-1)} = \sigma_k e_j, \quad j \in \left\{ \Delta^{(k)} \right\} \tag{B2}$$

where $\Delta^{(k)}$ is a subset of $\Delta^{(1)}$ (defined in (5)) containing the indices not already used in the construction of the matrices $M^{(1)}$, $M^{(2)}$, . . ., $M^{(k-1)}$. $M^{(k)}$ can be constructed if and only if $\sigma_k \neq 0$ (ref. 8).

(7) $Q^{(k-1)} = M^{(k-1)}, M^{(k-2)}, . . ., M^{(1)}$ \hfill (B3)

The algorithm is now accomplished by completing steps 1 to 4 for $k = 1, 2, \ldots, n$ and by computing the feedback gains from equation (12) using $C = I_n$.

Step 1:

For $\lambda = \lambda_k$, compute $\delta_k = \det\left[\lambda_k I_{n-m} - F\right]$

(a) if $\delta_k = 0$, perturb λ_k to $\left[\lambda_k + \epsilon\lambda_k\right]$ and repeat the calculation of δ_k.

(b) if $\delta_k \neq 0$, proceed to step 2.

Step 2:

Compute

$$D^{(k)} = \left[\lambda_k I_{n-m} - F\right]^{-1}\left[G + \lambda_k S\right] \tag{B4}$$

Step 3:

For some $j \in \left\{\Delta^{(k)}\right\}$,

(a) Compute $\left[g_j^{(k)}\right]' = f_j^{(k-1)} + h_j^{(k-1)}D^{(k)}$ where $\left[f_j^{(k-1)} : h_j^{(k-1)}\right]$ is the jth row of the transformation matrix $Q^{(k-1)}$; $g_j^{(k)} \in R^m$ and $h_j^{(k-1)} \in R^{1\times(n-m)}$.

(b) Compute

$$\sigma_k = \left[g_j^{(k)}\right]' z_k \tag{B5}$$

(i) if $\sigma_k \neq 0$, compute $w = D^{(k)}z_k$ and $M^{(k)}$ (from eq. (B2)) and return to step 1 for the next value of k.

(ii) if $\sigma_k = 0$, select another $j \in \left\{\Delta^{(k)}\right\}$ and return to step 3(a).

(iii) if $\sigma_k = 0$ for all $j \in \left\{\Delta^{(k)}\right\}$, go to step 4.

Step 4:

For some $j \in \left\{\Delta^{(k)}\right\}$,

(a) if $g_j^{(k)} \neq 0$, perturb z_k to $(z_k + \epsilon z_k)$ to make $\sigma_k \neq 0$, compute w_k and $M^{(k)}$, and return to step 1 for the next value of k.

(b) if $g_j^{(k)} = 0$, select another $j \in \left\{\Delta^{(k)}\right\}$ and return to step 4(a).

(c) if $g_j^{(k)} = 0$ for all $j \in \left\{\Delta^{(k)}\right\}$, perturb λ_k to $(\lambda_k + \epsilon\lambda_k)$ and return to step 1.

APPENDIX B

The following observations can be made regarding the algorithm as outlined.

     (i) Extension of the algorithm to assign complex eigenvalues is straight-forward (ref. 8).

     (ii) If some of the eigenvalues of Λ coincide with those of F, either a perturbation can be made in the eigenvalue specification as in step 1, or special eigenvector structures can be derived by noting that equation (6) has a solution for $z_i = 0$ (ref. 8).

     (iii) The iterative procedure in part ii of step 3(b) attempts to meet exact eigenvalue/eigenvector specifications. In step 4(b) an attempt is made to meet exact λ_k specifications with slightly relaxed z_k specifications. The test in step 4(c) indicates that the λ_k specification implied that the corresponding v_k lies in the eigenspace already generated. The test thus demands an eigenvalue perturbation.

     (iv) The σ_k provide a good measure of the linear independence between eigenvectors, if all vectors are normalized to a standard basis. Since the determinant of the modal matrix V is given by the product of the σ_k, $(k = 1, 2, \ldots, n)$, the numerical ill conditioning of V for inversion can be effectively controlled by setting a tolerance on the σ_k.

OUTPUT FEEDBACK EXTENSIONS

     Since the distribution matrix C is already in the special canonical form, generating linearly independent vectors to guarantee existence of feedback matrices in equations (12) and (14) is simply achieved by restricting the admissible pivotal indices $\Delta^{(k)}$ (eq. (B2)) in the spectral synthesis algorithm as follows.

Case I: Computing $\bar{K}$, Equation (12)

     In equation (5), let $\Delta_r^{(1)}$ be the set of r column indices corresponding to the r linearly independent columns of $C(I_r)$. Then apply the spectral synthesis algorithm using $\Delta_r^{(1)}$ instead of $\Delta^{(1)}$ (notation (5)) for $k = 1, 2, \ldots, r$. This application guarantees the invertibility of $CV^{(r)}$ in equation (12).

Case II: Computing $\tilde{K}$, Equation (14)

     In equation (5), let $\Delta_t^{(1)}$ be the set of t column indices not containing the vector c (eqs. (3)). Then apply the spectral synthesis algorithm using $\Delta_t^{(1)}$ instead of $\Delta^{(1)}$ (notation (5)) for $k = 1, 2, \ldots, t$. This application guarantees the nonsingularity of N_1 (eq. (13)). Further, since in this case CN is rank t, the feedback matrix $\tilde{K}$ (eq. (14)), computed using the relation

$$\tilde{K}CN = B_1^{-1}\left[Z^{(t)}\Lambda_t - A_1 N\right] \tag{B6}$$

25

to assign t eigenvalues, is not unique. However, a minimum norm least-squares solution for $\tilde{K}$ can be computed using the pseudo inverse of CN.
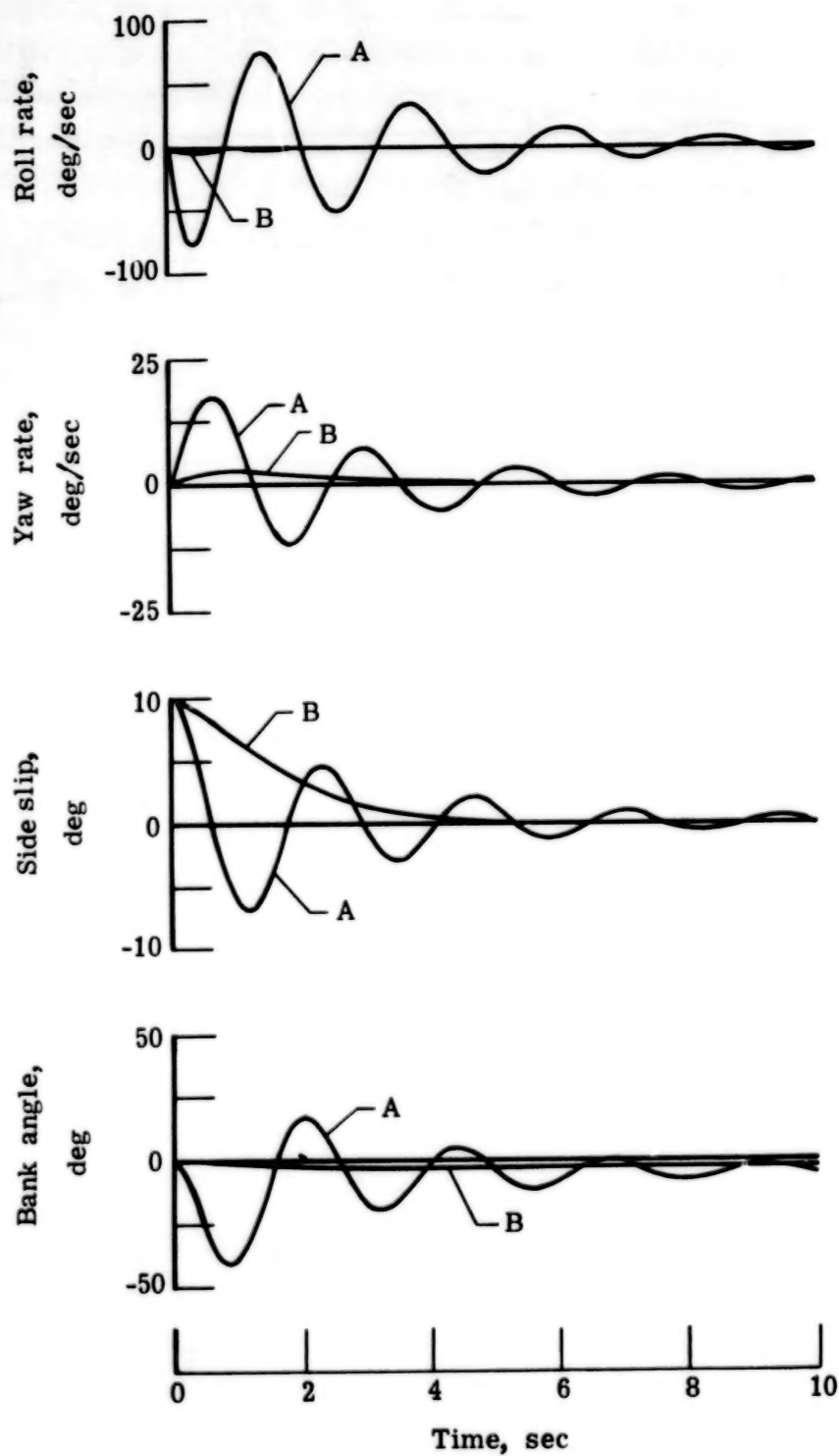
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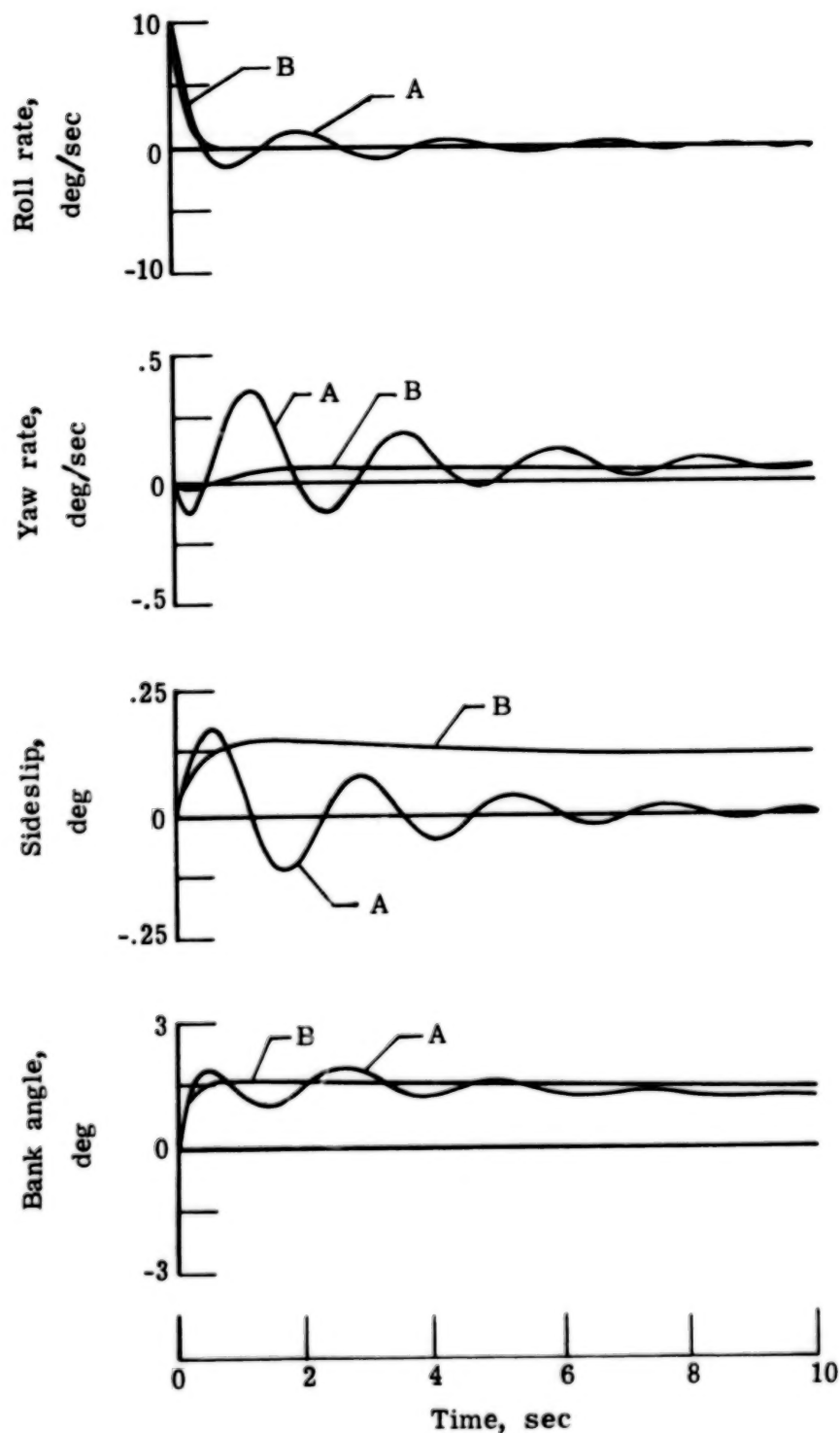
(a) p(0) = 10 deg/sec.

Figure 1.- Comparison of free aircraft and augmented aircraft response to roll
rate and sideslip step disturbances. A indicates free aircraft response
and B corresponds to augmented aircraft response.

28

(b) $\beta(0) = 10^{\circ}$.

Figure 1.- Concluded.

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16. Abstract
The problem of eigenvalue assignment in a linear time-invariant system using output feedback is considered. New sufficient conditions are derived to assign an almost arbitrary set of minimum $(n, m + r - 1)$ distinct eigenvalues where n, m, and r are the number of states, inputs, and outputs, respectively. These conditions precisely identify the class of systems where such an assignment is impossible. The synthesis technique also highlights the freedom in selection of closed-loop eigenvectors under output feedback. The utility of eigenvalue/eigenvector assignment in transient response shaping is illustrated by the design of a controller for the lateral dynamics of an aircraft.

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